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A Study on the Implementation of Lu-Chipman Decomposition Method via Matlab Simulation

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ABSTRACT – Polarization is a study of the property of the orientation of oscillations of the electromagnetic waves. Applications of polarization are vast from astronomy to photography. In this field, the property of all waves can be described using Stoke matrix, Mueller matrix and other several decomposition methods. One of them is the decomposition method suggested by Lu and Chipman by decomposing the Mueller Matrix into the Diattenuator matrix, M_D, Retarder matrix, M_R and Depolarizer matrix, M_Δ. This method is better known as Lu Chipman Decomposition. This paper experiments with this decomposition method by implementing it with several synthetic data.

1. INTRODUCTION

In 1995, S. Y. Lu and R. A. Chipman proposed a three factor's matrices decomposition of an arbitrary Mueller matrix, M based on a generalization of the polar decomposition to the depolarizing case which are a diattenuator, M_D, a retarder, M_R and a depolarizer, M_Δ. The objectives of this work are estimate Mueller matrix and decompose it into three "basis" are Retarder, Diattenuater and Depolarizer (for nondepolarize & depolarize).

$$M = M_\Delta \times M_R \times M_D \tag{1}$$

where a depolarizing matrix, M_D, accounts for the depolarizing effects of the medium, a retarder matrix, M_R, describes the effects of linear birefringence and optical activity, and a diattenuator matrix, M_Δ, includes the effects of linear and circular dichroism [3].

2. METHODOLOGY

In the original paper written by S.Y. Lu and R. A. Chipman [1], the authors stated that the decomposition of the Muller Matrix is done based on two cases: depolarizing Mueller matrix and non-depolarizing Mueller Matrix. The following subsections will describes these two cases.

2.1 Decomposition of Lu-Chipman Method for Depolarizing Mueller Matrix

Similar to the explanation earlier, the Depolarizing Mueller Matrix can be decomposed into three factors: a Diattenuator, followed by a Retarder, then followed by a purely Depolarizing factor with maintaining the order of the matrices. This as shown in Eq.(2). From Eq.(2), Eq.(3) can be obtained.

$$M' = MM_D^{-1} \tag{2}$$

where

$$M' = M_\Delta M_R \tag{3}$$

The Mueller matrix of the diattenuator can be calculated by Eq. (4)

$$M_D = \begin{bmatrix} 1 & \vec{D}^T \\ \vec{D} & m_D \end{bmatrix} \tag{4}$$

Given that m_D and $\vec{D}$ as describe in Eq.(5) and Eq.(6)

$$m_D = \sqrt{1-D^2}\,I + \left(1-\sqrt{1-D^2}\right)DD^T \tag{5}$$

$$D = \vec{D}/\left|\vec{D}\right| \tag{6}$$

where D is the diattenuation of the diattenuator and $\vec{D}$ is the diattenuation vector and they can be calculated directly from the Mueller matrix by Eq.(7) and Eq.(8)

$$D = \frac{1}{m_{00}}\sqrt{m_{01}^2 + m_{02}^2 + m_{03}^2} \tag{7}$$

$$\vec{D} = \frac{1}{m_{00}}\begin{bmatrix} m_{01} \\ m_{02} \\ m_{03} \end{bmatrix} \tag{8}$$

From the Mueller matrix of the diattenuator, we can find M' using Eq.(9)

$$M' = \begin{bmatrix} 1 & \vec{0}^T \\ \vec{0} & m' \end{bmatrix} = M_\Delta M_R \tag{9}$$

where m' as mentioned in Eq.(10).

$$m' = m_\Delta m_R \tag{10}$$

By finding the eigenvalues of $m'(m')^T$ which are λ_1, λ_2 and λ_3 then find m_Δ by Eq.(11).

$$m_\Delta = \pm\left[m'(m')^T + \left(\sqrt{\lambda_1\lambda_2} + \sqrt{\lambda_2\lambda_3} + \sqrt{\lambda_3\lambda_1}\right)I\right]^{-1} \times\left[\left(\sqrt{\lambda_1} + \sqrt{\lambda_2} + \sqrt{\lambda_3}\right)m'(m')^T + \sqrt{\lambda_1\lambda_2\lambda_3}I\right] \quad (11)$$

After that the Mueller matrix of the depolarizer is directly calculated from Eq.(12).

$$M_\Delta = \begin{bmatrix} 1 & \vec{0}^T \\ \vec{0} & m_\Delta \end{bmatrix} \quad (12)$$

From Eq.(3) and (12), the Mueller matrix of the retarder can be measure by Eq.(13).

$$M_R = M_\Delta^{-1}M' \quad (13)$$

2.2 Decomposition of Lu-Chipman Method for Nondepolarizing Mueller Matrix

For a non-depolarizing Mueller matrix, M , can be decomposes into a diattenuator, M_D and retarder, M_R . This relationship can be describes as Eq.(14).

$$M = M_R M_D \quad (14)$$

Given that the normalized M has a characteristic as Eq.(15).

$$M = \begin{bmatrix} 1 & \vec{D}^T \\ \vec{P} & m \end{bmatrix} \quad (15)$$

The (normalized) diatenuator matrix, M_D is describes as Eq.(16).

$$M_D = \begin{bmatrix} 1 & \vec{D}^T \\ \vec{D} & m_D \end{bmatrix} \quad (16)$$

where m_D is a submatrix in M_D that given by Eq.(17).

$$m_D = aI_3 + b\left(\vec{D}\cdot\vec{D}^T\right) \quad (17)$$

I_3 is the 3 × 3 identity matrix and a and b can derived using Eq.(18) and Eq.(19).

$$a = \sqrt{1-D^2} \quad (18)$$

$$b = \frac{1-a}{D^2} \quad (19)$$

D is the norm of $\vec{D}$ as shown in Eq.(20).

$$D = |\vec{D}| \quad (20)$$

The (normalized) retarder matrix, M_R can be obtained by Eq.(21).

$$M_R = \begin{bmatrix} 1 & \vec{0}^T \\ \vec{0} & m_R \end{bmatrix} \quad (21)$$

where m_R is a submatrix in M_R that given by Eq.(22).

$$m_R = \frac{1}{a}\left[m - b\left(\vec{P}\cdot\vec{D}^T\right)\right] \quad (22)$$

3. RESULTS AND DISCUSSION

The decomposition of Lu-Chipman for Depolarizing Mueller Matrix is been tested for simulated Mueller matrix obtained by multiplying a Diattenuator Mueller matrix by a Retarder Mueller matrix by a Depolarizer Mueller matrix. After applying Decomposition for Depolarizing Mueller matrix, we obtain back our original Diattenuator, Retarder and Depolarizer Mueller matrices. The same Mueller matrix is tested again small random noise at the range of [-0.01%, 0.01%] and [-0.1%, 0.1%]. The sums of the square error for the Mueller matrices in this cases are shown in Table 1 and Table 2.

Table 1 Depolarizing case simulation result.

Sum of Square Error (Average of 100 runs)				
Case	*Depolarizing*			
Random Range	*[-0.01%, 0.01%]*		*[-0.1%, 0.1%]*	
Medium	*Mean*	*Standard Deviation*	*Mean*	*Standard Deviation*
$\boldsymbol{M}$	0.00053	0.00014	0.05370	0.01150
$\boldsymbol{M_\Delta}$	0.00110	0.00038	0.06540	0.02130
$\boldsymbol{M_R}$	0.00052	0.00026	0.05330	0.02680
$\boldsymbol{M_D}$	0.00043	0.00025	0.04690	0.02420

Table 2 Non-depolarizing case simulation result.

Sum of Square Error (Average of 100 runs)				
Case	*Non-Depolarizing*			
Random Range	*[-0.01%, 0.01%]*		*[-0.01%, 0.01%]*	
Medium	*Mean*	*Mean*	*Mean*	*Mean*
$\boldsymbol{M}$	0.00052	0.00052	0.00052	0.00052
$\boldsymbol{M_\Delta}$	NA	NA	NA	NA
$\boldsymbol{M_R}$	0.00074	0.00074	0.00074	0.00074
$\boldsymbol{M_D}$	0.00048	0.00048	0.00048	0.00048

4. CONCLUSION

Lu-Chipman can be used to decompose the Mueller matrix into a Diattenuator, a Retarder, and a Depolarizer for Depolarizing Mueller Matrix and it can decompose the Nondepolarizing Mueller Matrix into a diattenuator, and retarder. We show that Lu-Chipman method can give us small sum of square error for Mueller matrix added to random noise and the error is increased by increasing the random noise in the overall Mueller matrix.

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